Electronic Journal of Qualitative Theory of Differential Equations
2012, No. **32**, 1-11; **http://www.math.u-szeged.hu/ejqtde/**

Dissipative Control for Singular Impulsive Dynamical Systems

Li Yang[a,b], Xinzhi Liu[b,1] and Zhigang Zhang[b,c]

a. College of Mathematics, Liaoning University, Shenyang, Liaoning, China, 110036

b. Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

c. Department of Statistics and Applied Mathematics, Hubei University of Economics, Wuhan, China

Abstract

The aim of this work is to study the dissipative control problem for singular impulsive dynamical systems. We start by introducing the impulse to the singular systems, and give the definition of the dissipation for singular impulsive dynamical systems. Then we discuss the dissipation of singular impulsive dynamical systems, we obtain some sufficient and necessary conditions for dissipation of these systems by solving some linear matrix inequalities (LMIs). By using this method, we design a state feedback controller to make the closed-loop system dissipative. At last, we testify the feasibility of the method by a numerical example.

Keywords: Dissipative control; singular dynamical impulsive systems; supply rate

1. Introduction

Singular systems are found in engineering systems, such as electrical circuit network, power systems, aerospace engineering, and chemical processing, social systems, economic systems, biological systems, network analysis, time-series analysis, singular singularly perturbed systems with which the singular system has a great deal to do, etc [1]. The concept of dissipation is of much interest in physics and engineering [2, 3, 4]. And dissipative systems can be used as models for physical phenomena in which the energy or entropy exchanged with the environment plays a role. The storage function measures the amount of energy which is stored inside the system at any instant of time, and when the storage function is taken as the special form, the dissipation is changed into passivity. In practical systems, such as in the electrical circuit network systems, the impulse always exists, thus many scholars interest in the control problem of dynamical

[1]Corresponding author

impulse systems, and this problem has been extensively studied and applied in many areas; see, e.g. [5, 6, 7, 8]. Now Hadded et al. have developed dissipation and exponential dissipation concepts for nonlinear impulse dynamical systems and left-continuous systems, see, e.g. [9, 10, 11, 12]. They have extended the notions of classical dissipative theory using generalized storage functions and supply rates for dynamical impulsive systems and left-continuous dynamical systems. And in [13], robust dissipativity for uncertain impulsive dynamical systems is discussed. Moreover singular systems and the impulse exists widely [14, 15], in order to study the dynamical impulsive systems more widely, we discuss the robust dissipative control problem for singular dynamical impulsive system , and obtain the robust impulsive dissipative state feedback controller of singular dynamical impulsive systems by solving the linear matrix inequalities.

The contents of the paper are as follows. In Section 2 we establish definitions, notation and review some important results. In Section 3, we obtain sufficient and necessary conditions of the dissipation for singular dynamical impulse systems, and then give the sufficient and necessary conditions to prove the singular dynamical impulsive system is dissipative. In Section 4, by using the method we design a state feedback controller for singular dynamical impulsive systems. Then in Section 5, we testify the feasibility of the method by a numerical example. Finally we draw conclusions in Section 6.

2. Singular Impulsive Dynamical Systems

In this section we introduce the dynamical impulse into the singular systems, and a singular dynamical impulsive system consists of three elements:

(i) A singular continuous-time dynamical equation, which governs the motion of the system between resetting events;
(ii) A difference equation, which governs the way the states are instantaneously changed when a resetting event occurs;
(iii) A criterion for determining when the states of the system are to be reset.

A singular dynamical impulsive system has the following form

$$\begin{cases} E\dot{x}(t) = Ax(t) + B_c\omega_c(t), \quad t \neq t_k; \\ \Delta x(t) = x(t_k^+) - x(t_k) = D_k x(t) + B_d\omega_d(t), \quad t = t_k; \\ y_c(t) = C_c x(t) + D_c\omega_c(t), \quad t \neq t_k; \\ y_d(t) = C_d x(t) + D_d\omega_d(t), \quad t = t_k. \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state, $\omega_c \in \mathbf{R}^{m_c}$, $\omega_d \in \mathbf{R}^{m_d}$ the outside perturbation of the system whose square is integral, and $y_c \in \mathbf{R}^{l_c}$, $y_d \in \mathbf{R}^{l_d}$ are output, E, A, B_c, B_d, D_k, C_c, C_d, D_c

and D_d are known constant matrices of appropriate dimensions, and E is singular.

Definition 1. A function $r_c(\omega_c, y_c), r_d(\omega_d, y_d)$, where $r_c : \mathbf{R}^{m_c} \times \mathbf{R}^{l_c} \to \mathbf{R}$ and $r_d : \mathbf{R}^{m_d} \times \mathbf{R}^{l_d} \to \mathbf{R}$ are such that $r_c(0,0) = 0$ and $r_d(0,0) = 0$, is called a supply rate of system (1) if $r_c(\omega_c, y_c)$ is locally integrable; that is, for all input-output pairs $(\omega_c(t), y_c(t))$, $r_c(\omega_c, y_c)$ satisfies $\int_t^{\hat{t}} |r_c(\omega_c(s), y_c(s))| ds < \infty$ for any $\hat{t} \geq t \geq 0$, and $r_d(\omega_d, y_d)$ is locally summable. In other words, for all input-output pairs $(\omega_d(t_k), y_d(t_k)), r_d(\omega_d, y_d)$ satisfies

$$\sum_{k \in \mathbb{N}[t, \hat{t})} |r_d(\omega_d(t_k), y_d(t_k))| < \infty$$

where $\mathbb{N}[t, \hat{t}) = \{k : t \leq t_k < \hat{t}\}$.

Definition 2. The singular impulsive dynamical system (1) is said to be dissipative with respect to supply rate (r_c, r_d) if there exists a $C^r(r \geq 0)$ nonnegative function $V : \mathbf{R}^n \to \mathbf{R}$ with $V(0) = 0$, called storage function, such that, for all $(\omega_c, \omega_d) \in U$, the following dissipation inequality holds:

$$V(x(t)) \leq V(x(t_0)) + \int_{t_0}^{t} r_c(\omega_c(s), y_c(s)) ds + \sum_{k \in \mathbb{N}[t, \hat{t})} r_d(\omega_d(t_k), y_d(t_k)) \tag{2}$$

where $x(t)$, $t \geq t_0$ is a solution to system (1) with $(\omega_c, \omega_d) \in \mathbf{R}^{m_c} \times \mathbf{R}^{m_d}$, and $x(t_0) = x_0$.

Lemma 1. *[16] Suppose S_{11} and S_{22} are symmetric. The condition*

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix} \geq 0 \tag{3}$$

is equivalent to

$$S_{22} \geq 0, \quad S_{11} - S_{12} S_{22}^{+} S_{12}^{\mathrm{T}} \geq 0, \quad S_{12}(I - S_{22} S_{22}^{+}) = 0, \tag{4}$$

where S_{22}^{+} denotes the Moore-Penrose inverse of S_{22}.

3. Dissipation of Singular Impulsive Dynamical Systems

From [9], we can get the trivial result of singular impulsive dynamical system as the following theorem:

Theorem 1. *A singular impulsive dynamical system given by (1) is dissipative with respect to the supply rate (r_c, r_d) if and only if there exists a C^1 nonnegative definite function*

$V : \mathbf{R}^n \rightarrow \mathbf{R}$ *such that for all* $k \in N = \{0, 1, 2, \ldots\}$

$$V(x(\hat{t})) - V(x(t)) \leq \int_t^{\hat{t}} r_c(\omega_c(s), y_c(s))ds, \qquad t_k < t \leq \hat{t} \leq t_{k+1} \tag{5}$$

$$V(x(t_k^+)) - V(x(t_k)) \leq r_d(\omega_d(t_k), y_d(t_k)). \tag{6}$$

Remark 1. In the theorem, $V(\cdot, x(\cdot))$ is C^1 a.e. on $[t_0, \infty)$ except on an unbounded closed discrete set $\mathfrak{T} = \{t_1, t_2, \cdots\}$, where $\mathfrak{T}$ is the set of times when jumps occur for $x(t)$, then an equivalent statement for dissipation of the singular impulsive dynamical system (1) with respect to the supply rate (r_c, r_d) is

$$\dot{V}(t, x(t)) \leq r_c(\omega_c(t), y_c(t)), \qquad t_k < t \leq t_{k+1} \tag{7}$$

$$\begin{aligned} \Delta V(t_k, x(t_k)) &= V(t_k^+, x(t_k^+)) - V(t_k, x(t_k)) \\ &\leq r_d(\omega_d(t_k), y_d(t_k)). \end{aligned} \tag{8}$$

In the paper, we consider the following storage function and supply rate

$$V = x^{\mathrm{T}} E^{\mathrm{T}} P x \tag{9}$$

$$\begin{aligned} r_c(\omega_c, y_c) &= y_c^{\mathrm{T}} Q_c y_c + 2 y_c^{\mathrm{T}} S_c \omega_c + \omega_c^{\mathrm{T}} R_c \omega_c, \\ r_d(\omega_d, y_d) &= y_d^{\mathrm{T}} Q_d y_d + 2 y_d^{\mathrm{T}} S_d \omega_d + \omega_d^{\mathrm{T}} R_d \omega_d, \end{aligned} \tag{10}$$

where $E^{\mathrm{T}} P = P^{\mathrm{T}} E \geq 0$, and the matrices Q_c, S_c, R_c, Q_d, S_d, and R_d are given matrices with appropriate dimensions.

Without loss of the generality, we can assume that Q_c and Q_d are both negative definite. Then we will discuss the dissipation of the singular dynamical impulsive systems.

Theorem 2. *For system* (1)*, if and only if the following inequalities have the feasible solution* P*,*

$$\begin{bmatrix} A^{\mathrm{T}} P + P^{\mathrm{T}} A & P^{\mathrm{T}} B_c - C_c^{\mathrm{T}} S_c & C_c^{\mathrm{T}} \\ * & -D_c^{\mathrm{T}} S_c - S_c^{\mathrm{T}} D_c - R_c & D_c^{\mathrm{T}} \\ * & * & Q_c^{-1} \end{bmatrix} \leq 0 \,, \tag{11}$$

$$\begin{bmatrix} -E^{\mathrm{T}} P & -C_d^{\mathrm{T}} S_d & (I + D_k)^{\mathrm{T}} & C_d^{\mathrm{T}} \\ * & -D_d^{\mathrm{T}} S_d - S_d^{\mathrm{T}} D_d - R_d & B_d^{\mathrm{T}} & D_d^{\mathrm{T}} \\ * & * & -(E^{\mathrm{T}} P)^{+} & 0 \\ * & * & * & Q_d^{-1} \end{bmatrix} \leq 0 \,, \tag{12}$$

$$(I + D_k)(I - E^+E) = 0\ , \tag{13}$$

$$B_d^{\mathrm{T}}(I - E^+E) = 0\ , \tag{14}$$

system (1) *is dissipative with respect to the supply rate* (10)*, where* " $*$ " *denotes the symmetric part of the matrix.*

Proof. According to the storage function and the supply rate, we obtain

$$\dot{V}(t, x(t)) = \begin{bmatrix} x \\ \omega_c \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} A^{\mathrm{T}}P + P^{\mathrm{T}}A & P^{\mathrm{T}}B_c \\ B_c^{\mathrm{T}}P & 0 \end{bmatrix} \begin{bmatrix} x \\ \omega_c \end{bmatrix}, \tag{15}$$

$$r_c(\omega_c, y_c) = \begin{bmatrix} x \\ \omega_c \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} C_c^{\mathrm{T}}Q_cC_c & C_c^{\mathrm{T}}Q_cD_c + C_c^{\mathrm{T}}S_c \\ D_c^{\mathrm{T}}Q_c^{\mathrm{T}}C_c + S_c^{\mathrm{T}}C_c & D_c^{\mathrm{T}}Q_cD_c + D_c^{\mathrm{T}}S_c + S_c^{\mathrm{T}}D_c + R_c \end{bmatrix} \begin{bmatrix} x \\ \omega_c \end{bmatrix}, \tag{16}$$

and

$$\begin{aligned} & V(t_k^+, x(t_k^+)) - V(t_k, x(t_k)) \\ & = x^{\mathrm{T}}(t_k^+)E^{\mathrm{T}}Px(t_k^+) - x^{\mathrm{T}}(t_k)E^{\mathrm{T}}Px(t_k) \\ & = \begin{bmatrix} x(t_k) \\ \omega_d(t_k) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} (I + D_k)^{\mathrm{T}}E^{\mathrm{T}}P(I + D_k) - E^{\mathrm{T}}P & (I + D_k)^{\mathrm{T}}E^{\mathrm{T}}PB_d \\ B_d^{\mathrm{T}}E^{\mathrm{T}}P(I + D_k) & B_d^{\mathrm{T}}E^{\mathrm{T}}PB_d \end{bmatrix} \begin{bmatrix} x(t_k) \\ \omega_d(t_k) \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} & r_d(\omega_d(t_k), y_d(t_k)) \\ & = \begin{bmatrix} x(t_k) \\ \omega_d(t_k) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} C_d^{\mathrm{T}}Q_dC_d & C_d^{\mathrm{T}}Q_dD_d + C_d^{\mathrm{T}}S_d \\ D_d^{\mathrm{T}}Q_d^{\mathrm{T}}C_d + S_d^{\mathrm{T}}C_d & D_d^{\mathrm{T}}Q_dD_d + D_d^{\mathrm{T}}S_d + S_d^{\mathrm{T}}D_d + R_d \end{bmatrix} \begin{bmatrix} x(t_k) \\ \omega_d(t_k) \end{bmatrix}, \end{aligned}$$

then by Theorem 1, we obtain

$$\begin{bmatrix} A^{\mathrm{T}}P + P^{\mathrm{T}}A - C_c^{\mathrm{T}}Q_cC_c & P^{\mathrm{T}}B_c - C_c^{\mathrm{T}}Q_cD_c - C_c^{\mathrm{T}}S_c \\ * & -D_c^{\mathrm{T}}Q_cD_c - D_c^{\mathrm{T}}S_c - S_c^{\mathrm{T}}D_c - R_c \end{bmatrix} \leq 0 \tag{17}$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix} \leq 0 \tag{18}$$

where

$$\Xi_{11} = (I+D_k)^{\mathrm{T}}E^{\mathrm{T}}P(I+D_k) - E^{\mathrm{T}}P - C_d^{\mathrm{T}}Q_dC_d,$$

$$\Xi_{12} = (I+D_k)^{\mathrm{T}}E^{\mathrm{T}}PB_d - C_d^{\mathrm{T}}Q_dD_d - C_d^{\mathrm{T}}S_d,$$

$$\Xi_{22} = B_d^{\mathrm{T}}E^{\mathrm{T}}PB_d - D_d^{\mathrm{T}}Q_dD_d - D_d^{\mathrm{T}}S_d - S_d^{\mathrm{T}}D_d - R_d.$$

Then using Lemma 1, we get the the conclusion of theorem. □

4. Dissipative Controller Design for Singular Impulsive Dynamical Systems

In this section, we will design a state feedback controller for the singular dynamical impulsive systems. The system considered is as the following:

$$\begin{cases} E\dot{x}(t) = Ax(t) + B_c\omega_c(t) + G_cu_c(t), \quad t \neq t_k; \\ \Delta x(t) = x(t_k^+) - x(t_k) = D_kx(t) + B_d\omega_d(t) + G_du_d(t), \quad t = t_k; \\ y_c(t) = C_cx(t) + D_c\omega_c(t) + J_cu_c(t), \quad t \neq t_k; \\ y_d(t) = C_dx(t) + D_d\omega_d(t) + J_du_d(t), \quad t = t_k. \end{cases} \tag{19}$$

where $u_c \in \mathbf{R}^{s_c}$, $u_d \in \mathbf{R}^{s_d}$ are the input. And the state feedback controller is as the following:

$$\begin{cases} u_c(t) = K_cx(t), \\ u_d(t_k) = K_dx(t_k). \end{cases} \tag{20}$$

Using Theorem 2 , we obtain the following results:

Theorem 3. *For system* (1), *if* $B_d^{\mathrm{T}}[I-(E^{\mathrm{T}})^+E] = 0$ *and the following inequalities have the feasible solution* X, W_c, W_d,

$$X^{\mathrm{T}}E^{\mathrm{T}} = EX \geq 0 \tag{21}$$

$$\begin{bmatrix} (AX+G_cW_c)^{\mathrm{T}} + (AX+G_cW_c) & B_c - (C_cX+J_cW_c)^{\mathrm{T}}S_c & (C_cX+J_cW_c)^{\mathrm{T}} \\ * & -D_c^{\mathrm{T}}S_c - S_c^{\mathrm{T}}D_c - R_c & D_c^{\mathrm{T}} \\ * & * & Q_c^{-1} \end{bmatrix} \leq 0\ , \tag{22}$$

$$\begin{bmatrix} -X^{\mathrm{T}}E^{\mathrm{T}} & -(C_dX+J_dW_d)^{\mathrm{T}}S_d & (X+D_kX+G_dW_d)^{\mathrm{T}} & (C_dX+J_dW_d)^{\mathrm{T}} \\ * & -D_d^{\mathrm{T}}S_d - S_d^{\mathrm{T}}D_d - R_d & B_d^{\mathrm{T}} & D_d^{\mathrm{T}} \\ * & * & -X^{\mathrm{T}}(E^{\mathrm{T}})^+ & 0 \\ * & * & * & Q_d^{-1} \end{bmatrix} \leq 0\ , \tag{23}$$

$$(X+D_kX+G_dW_d)^{\mathrm{T}}[I-(E^{\mathrm{T}})^+E] = 0 \tag{24}$$

then system (1) *has dissipative feedback controller* (20)*, where* $K_c = W_cX^{-1}$*, and* $K_d = W_dX^{-1}$.

Proof. The closed-loop system of system (19) via the feedback controller (20) is

$$\begin{cases} E\dot{x}(t) = (A + G_cK_c)x(t) + B_c\omega_c(t), \quad t \neq t_k; \\ \Delta x(t) = x(t_k^+) - x(t_k) = (D_k + G_dK_d)x(t) + B_d\omega_d(t), \quad t = t_k; \\ y_c(t) = (C_c + J_cK_c)x(t) + D_c\omega_c(t), \quad t \neq t_k; \\ y_d(t) = (C_d + J_dK_d)x(t) + D_d\omega_d(t), \quad t = t_k. \end{cases} \tag{25}$$

Using Theorem 2, we obtain that

$$\begin{bmatrix} \bar{A}^{\mathrm{T}}P + P^{\mathrm{T}}\bar{A} & P^{\mathrm{T}}B_c - \bar{C}_c^{\mathrm{T}}S_c & \bar{C}_c^{\mathrm{T}} \\ * & -D_c^{\mathrm{T}}S_c - S_c^{\mathrm{T}}D_c - R_c & D_c^{\mathrm{T}} \\ * & * & Q_c^{-1} \end{bmatrix} \leq 0\,, \tag{26}$$

$$\begin{bmatrix} -E^{\mathrm{T}}P & -\bar{C}_d^{\mathrm{T}}S_d & (I + D_k + G_dK_d)^{\mathrm{T}} & \bar{C}_d^{\mathrm{T}} \\ * & -D_d^{\mathrm{T}}S_d - S_d^{\mathrm{T}}D_d - R_d & B_d^{\mathrm{T}} & D_d^{\mathrm{T}} \\ * & * & -(E^{\mathrm{T}}P)^{+} & 0 \\ * & * & * & Q_d^{-1} \end{bmatrix} \leq 0\,, \tag{27}$$

$$(I + D_k + G_dK_d)^{\mathrm{T}}[I - (E^{\mathrm{T}}P)^{+}(E^{\mathrm{T}}P)] = 0 \tag{28}$$

$$B_d^{\mathrm{T}}[I - (E^{\mathrm{T}}P)^{+}(E^{\mathrm{T}}P)] = 0 \tag{29}$$

where

$$\bar{A} = A + G_cK_c,$$

$$\bar{C}_c = C_c + J_cK_c,$$

$$\bar{C}_d = C_d + J_dK_d,$$

then multiply $diag\{X^{\mathrm{T}}, I\ , I\}$ by the left side of (26) and its transfer by the right side, and then using Lemma 1, we can obtain (22), where $X = P^{-1}$. And by using the similar method, we can obtain the result. □

5. Example

The parameters of system (1) are given as follows:

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\quad A=\begin{bmatrix}-10&0&0\\0&-15&0\\0&-10&-1\end{bmatrix},\quad B_c=\begin{bmatrix}-1&10\\-10&-7\\0&0\end{bmatrix},$$

$$B_d=\begin{bmatrix}-7&0\\0&-9\\0&0\end{bmatrix},\quad C_c=\begin{bmatrix}5&0&0\\0&-3&2\end{bmatrix},\quad C_d=\begin{bmatrix}1&0&0\\0&-2&2\end{bmatrix},\quad D_c=\begin{bmatrix}2&2\\1&3\end{bmatrix},$$

$$D_d=\begin{bmatrix}3&2\\0&1\end{bmatrix},\quad D_k=\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix},\quad G_c=\begin{bmatrix}-5&-10\\-4&-6\\-1&-10\end{bmatrix},\quad G_d=\begin{bmatrix}-1&1\\0&0\\0&0\end{bmatrix},$$

$$J_c=\begin{bmatrix}-9&5\\-6&7\end{bmatrix},\quad J_d=\begin{bmatrix}-12&13\\-1&-11\end{bmatrix},\quad Q_c=Q_d=\begin{bmatrix}-0.5&0\\0&-0.5\end{bmatrix},$$

$$S_c=\begin{bmatrix}7&2\\2&4\end{bmatrix},\quad S_d=\begin{bmatrix}2&1\\1&4\end{bmatrix},\quad R_c=\begin{bmatrix}10&0\\0&10\end{bmatrix},\quad R_d=\begin{bmatrix}17&0\\0&15\end{bmatrix}.$$

By using the Theorem 2 and Theorem 3, we can solve that

$$K_c=\begin{bmatrix}1.0679&0.4679&-0.3064\\0.9235&0.8445&-0.5522\end{bmatrix},\quad K_d=\begin{bmatrix}0.0620&-0.2048&0.1858\\-0.0155&-0.1812&0.1695\end{bmatrix}.$$

Therefore we can obtain the dissipative feedback controller is as follows:

$$u_c(t)=\begin{bmatrix}1.0679&0.4679&-0.3064\\0.9235&0.8445&-0.5522\end{bmatrix}x(t),$$

$$u_d(t_k)=\begin{bmatrix}0.0620&-0.2048&0.1858\\-0.0155&-0.1812&0.1695\end{bmatrix}x(t_k).$$

If we take

$$\omega_c(t)=\begin{bmatrix}0.5sint\\0.5sint\end{bmatrix},\quad \omega_d(t)=\begin{bmatrix}0.5cost\\0.5cost\end{bmatrix},$$

and the initial value is

$$x(0)==\begin{bmatrix}3\\-2\\1\end{bmatrix},$$

we get the value of $\dot{V}(t,x(t))-r_c(\omega_c(t),y_c(t))\leq 0,(\ t\neq t_k)$ for the system via the state feedback controller as Figure 1, and the value of $V(t_k^+)-V(t_k)-r_d(\omega_d(t),y_d(t))\leq 0,(\ t=t_k)$ for the system via the state feedback controller as Figure 2. From the figures, we can make a conclusion that a singular impulsive dynamical system is dissipative with respect

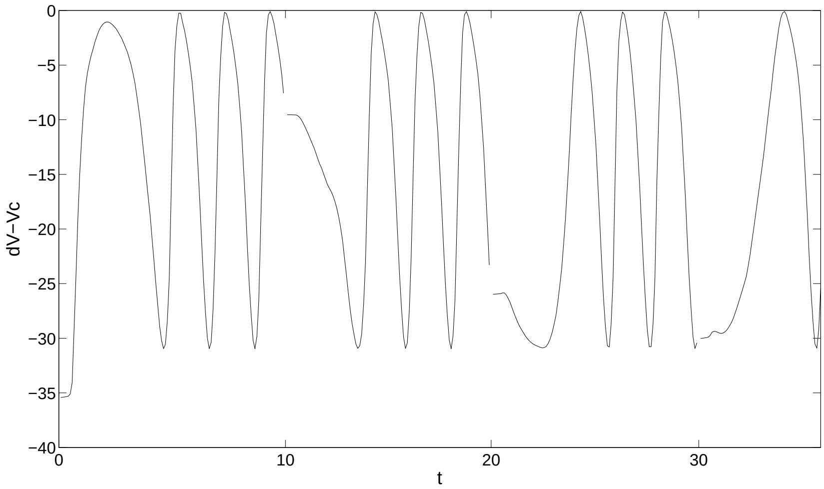

Figure 1: The value of $\dot{V}(t, x(t)) - r_c(\omega_c(t), y_c(t)) \leq 0, (\ t \neq t_k)$ for the system via the state feedback controller.

to the supply rate.

6. Conclusion

We have studied the dissipation with respect to the quadratic supply rate for singular dynamical impulsive systems. By solving linear matrix inequalities, some sufficient and necessary conditions of dissipation for this kind of system are obtained. As the problem of the dissipation with respect to other supply rate for the singular dynamical impulsive systems exists, we will discuss it in future papers.

Acknowledgment

The research of this paper was carried out while the first author was a post doctoral fellow at the University of Waterloo.

References

[1] L. Dai. *Singular Control Systems*. Springer-Verlag, USA, 1989.

[2] H.L. Trentelman and J.C. Willems. Every storage function is a state function. *Systems and Control letters*, 32(5):249–260, 1997.

[3] D. Hill and P. Moylan. The stability of nonlinear dissipative systems. *IEEE Transactions on Automatic Control*, 21(5):708–711, 1976.

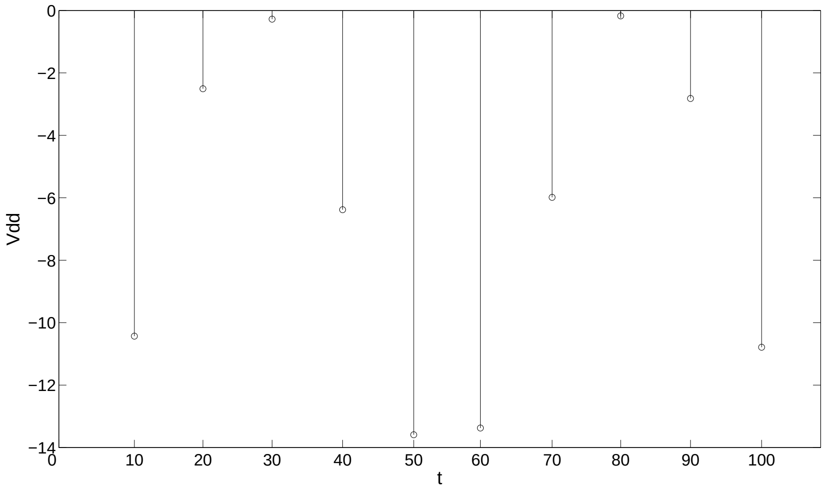

Figure 2: The value of $V(t_k^+) - V(t_k) - r_d(\omega_d(t), y_d(t)) \le 0, (\ t = t_k)$ for the system via the state feedback controller.

[4] X. Liu. Stability results for impulsive differential systems with applications to population growth models. *Dynamical Systems*, 9(2):163–174, 1994.

[5] C.J. Wang. State feedback impulse elimination of linear time-varying singular systems. *Automatica*, 32(1):133–136, 1996.

[6] S. Xu and C. Yang. A Matrix Inequalities Approach to the Robust Stability and Robust Stabilization for Uncertain Generalized Systems. *Acta Automatica Sinica*, 26(1):132–135, 2000.

[7] D.J. Hill and P.J. Moylan. Dissipative dynamical systems: Basic input-output and state properties. *J. Franklin Inst*, 309(5):327–357, 1980.

[8] C.J. Wang and H.E. Liao. Impulse observability and impulse controllability of linear time-varying singular systems. *Automatica*, 37(11):1867–1872, 2001.

[9] W.M. Haddad, V.S. Chellaboina, and N.A. Kablar. Nonlinear impulsive dynamical systems. I. Stability anddissipativity. In *Decision and Control, 1999. Proceedings of the 38th IEEE Conference on*, volume 5, 1999.

[10] J.C. Willems. Dissipative dynamical systems part II: Linear systems with quadratic supply rates. *Archive for Rational Mechanics and Analysis*, 45(5):352–393, 1972.

[11] H. Ye, A.N. Michel, and L. Hou. Stability theory for hybrid dynamical systems. *IEEE Transactions on Automatic Control*, 43(4):461–474, 1998.

[12] J. Shen and Z. Jing. Stability Analysis for Systems with Impulse Effects. *International Journal of Theoretical Physics*, 45(9):1703–1717, 2006.

[13] B. Liu, X. Liu, and X. Liao. Robust dissipativity for uncertain impulsive dynamical systems. *Mathematical Problems in Engineering*, 3(2003):119–128, 2003.

[14] V. Lakshmikantham and X. Liu. Stability criteria for impulsive differential equations in terms of two measures. *J. Math. Anal. Appl*, 137(2):591–604, 1989.

[15] W.M. Haddad and V. Chellaboina. Dissipativity theory and stability of feedback interconnections forhybrid dynamical systems. In *American Control Conference, 2000. Proceedings of the 2000*, volume 4, 2000.

[16] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan. *Linear matrix inequalities in system and control theory*. Society for Industrial Mathematics, 1994.

(Received October 5, 2011)